Optimization Criteria for Public Transportation Services

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Abstract

This paper describes precise criteria for public transportation services which can be used for (automated) time table optimization. All proposed evaluations are normalized to compare different trips and sets of routes even between different transport systems. The proposed functions are applied to an example with a periodic train schedule.

1 Introduction

Most existing approaches for optimization of public transportation services focus on the line optimization problem rather than on the time table itself (see [Hen76], [Hei88], [ZBKW97]). However, this can result in a line plan that doesn't allow a "good" time table. Present approaches that focus on time table optimization are based on given routes and stops where trains are supposed to wait for each other (see [Bra93], [BWZ97]). Nevertheless, it cannot be guaranteed that mean travel times will be reduced (travel times may increase due to waiting for other trains and people may benefit when changing trains).

The aim of this paper is to develop a formalism for optimization criteria and to show possibilities and variants of objective functions regarding automated time table optimization for public transportation services. In this approach lines are already given, so a combination with known line optimization algorithms is recommended.

First, single trips are evaluated, and then we get a valuation of a route as a weighted sum of these values. This will result in a ranking of the entire public transportation service. These values can be used to determine the quality of time tables or to compare transportation services in different towns. All values are normalized to $[1\ldots\infty]$, and value x can be read as "x times worse than the optimum".

For the sake of simplicity, only additive weight functions will be considered for optimization.

2 Definitions

Let S be a set of *locations* (stops, platforms), $\mathcal{N}_x \subseteq S \times S$ the *network* for mobile objects of type $x \in \{\text{ped(estrian), tram, bus, bike, } \ldots\}$ with $\mathcal{N}_x = \{(u,v) \mid \text{edge } (u,v) \text{ can be passed by mobile object } x\}$ and $T \equiv \mathbb{R}$ the set of *times*.

Let $\mathcal{L}_{tram} \subseteq S \times S^* \times S$ be the set of tram*lines* such that for any $l = (s_1, s_2, \ldots, s_{|l|}) \in \mathcal{L}_{tram}$ it holds $(s_i, s_{i+1}) \in \mathcal{N}_{tram}$ $(i = 1, \ldots, |l|-1)$.

The *time table* $\mathcal{T}_{tram} \subseteq S \times S \times \mathcal{L}_{tram} \times T \times T$ of $\mathcal{N}_{tram}$ is a set of 5-tuples

$$\mathcal{T}_{tram} = \{(s,d,l,t_d,t_a) \mid \text{the tram of line } l \text{ departs from source } s \in S \text{ at time } t_d \in T \text{ and arrives at destination } d \in S \text{ at time } t_a \in T\}$$

with appropriate constraints; e. g.: if $(s,d,l,t_d,t_a) \in \mathcal{T}_{tram}$ then $t_d < t_a$ and $t_a - t_d$ corresponds to the time needed for going from stop s to stop d, and $\mathcal{T}_{tram}$ is transitive, i. e. if $(s_1, s_2, l, t_d, t_{s_2})$, $(s_2, s_3, l, t_{s_2} + \Delta t, t_a) \in \mathcal{T}_{tram}$ then $(s_1, s_3, l, t_d, t_a) \in \mathcal{T}_{tram}$ and Δt is the delay at station s_2.

The set of *footpath travel times* $\mathcal{T}_{ped} \subseteq S \times S \times T$ of $\mathcal{N}_{ped}$ is given by $\mathcal{T}_{ped} = \{(s,d,t_{fp}) \mid \text{edge } (s,d) \text{ is walked through in time } t_{fp}\}$.

Time tables or travel times for other means of transport (e. g. $\mathcal{L}_{bus}/\mathcal{T}_{bus}$) may be given in a suitable way. Criteria to be used (time dependend or not) have to be given conveniently.

Given $\mathcal{T}_{ped}$ and the time tables for means of local transport $\mathcal{T}_{pub} = \bigcup_x \mathcal{T}_x$, $x \in \{\text{tram, bus, } \ldots\}$ we define the set of all (valid) *trips* $\mathcal{C} \subseteq \mathcal{T}_{pub} \times (\mathcal{T}_{ped} \times \mathcal{T}_{pub})^*$.

$$\begin{aligned} C := \big\{(c_0, c_1, \ldots, c_{2n}) \in \mathcal{T}_{pub} \times (\mathcal{T}_{ped} \times \mathcal{T}_{pub})^n \mid \\ n \geq 0,\ c_i.d = c_{i+1}.s,\ i = 0, \ldots, 2n-1, \\ c_i.t_a + c_{i+1}.t_{fp} \leq c_{i+2}.t_d,\ i = 0, 2, \ldots, 2n-2\big\} \end{aligned}$$

Changing trains always implies a footpath (insert a zero length footpath if platforms are the same), this is done to allow a more simple formalism in the evaluations below.

3 Optimization Criteria

There are some simple criteria from passenger's point of view

- travel time
- availability

- fare
- comfort

For sake of simplicity it is assumed that all passengers have the same preferences.

3.1 Evaluation of a trip

Optimization criteria can be combined in a weighted sum of travel time, number of changes, costs, etc. and result in a (weighted) subjective travel time [Hei88], [Len97], [Cla98], [Cla99]. The evaluation proposed here will differ because we are using a time table whereas [Hei88], [Len97] approximate waiting times using line frequencies. We only consider the following criteria (named by the respective weighting parameter γ_{xx}):

γ_{tt}: Travel times (in means of transport) – These times can be taken from the time tables $\mathcal{T}_{tram}, \mathcal{T}_{bus}, \ldots$

γ_{fp}: Footpath times when changing trams – These times are known from $\mathcal{T}_{ped}$.

γ_{wp}: Waiting periods when changing trams – These times can be determined from departure, arrival, and footpath times.

γ_{lc}: Loss of comfort – This is considered by a constant penalty for each change of trams.

γ_{fr}: Fare rating – It's assumed that the passenger is willing to spend more money, provided he reaches his destination earlier.

Other criteria to be considered similarly include footpath times to and from the stop, other comfort criteria (convenience of trams compared with busses), and so on.

Now we define the subjective travel time E_{subj} for a trip $C = (c_0, c_1, \ldots, c_{2n}) \in \mathcal{C} \subseteq \mathcal{T}_{pub} \times (\mathcal{T}_{ped} \times \mathcal{T}_{pub})^*$ with fare f_C:

$$\begin{aligned} E_{\mathrm{subj}}(c_0, \ldots, c_{2n}) := & \gamma_{\mathrm{tt}} \cdot \sum_{i=0,\ i \text{ even}}^{2n} (c_i.t_a - c_i.t_d) + \gamma_{\mathrm{fp}} \cdot \sum_{i=1,\ i \text{ odd}}^{2n-1} c_i.t_{fp} \\ & + \gamma_{\mathrm{wp}} \cdot \sum_{i=0,\ i \text{ even}}^{2n-2} \big(c_{i+2}.t_d - (c_i.t_a + c_{i+1}.t_{fp})\big) \\ & + \gamma_{\mathrm{lc}} \cdot n + \gamma_{\mathrm{fr}} \cdot f_C \end{aligned}$$

Given source u, destination v and departure time t we define

$$\begin{aligned} E_{\mathrm{subj\_min}}(u, v, t) := \min \big\{ & E_{\mathrm{subj}}(C) \mid C = (c_0, \ldots, c_m) \in \mathcal{C}, \\ & c_0.s = u, c_m.d = v, c_0.t_d = t \big\} \end{aligned}$$

To compare different trips we normalize these values relative to the minimal (subjective) travel time E_{opt} from u to v at departure time t

(minimizing over all possible time tables). This also solves the problem of overweighting long routes when evaluating public transportation services in total.

$$E_{\text{opt}}(u,v,t) := \min_{\mathcal{T}_{pub}}\{E_{\text{subj\_min}}(u,v,t) \text{ when using time table } \mathcal{T}_{pub}\}$$

E_{opt} may be time-dependend because travel times may vary with respect to the starting time (e. g. busses). Other means of transport like bike, car, etc. can also be regarded here. Restricted to public transport we can assume that the optimum is achieved if a tram goes directly from the source to the destination (speed-limits have to be taken into account and the fare is properly chosen as cheap as possible for trips from u to v). Therefore it is possible to compute $E_{\text{opt}}(u,v,t)$.

We define the *evaluation of a trip* as

$$E_{\text{trip}}(u,v,t) := \frac{E_{\text{subj\_min}}(u,v,t)}{E_{\text{opt}}(u,v,t)}$$

This quotient will be 1 if $\mathcal{T}_{pub}$ is optimal with respect to this trip.

3.2 Evaluation of a route

Let $D_{u,v}(t)$ be the (aggregated) volume of traffic on (all) route(s) from u to v, i. e., $\int_{t_1}^{t_2} D_{u,v}(t)\mathrm{d}t$ is the expected number of persons who want to travel from u to v during the time interval from t_1 to t_2.

For optimization we fix a period of time *start* to *end* that is to be used in all evaluations below.

3.2.1 Extended evaluation of a trip

People coming to a stop will have to wait for the *next trip*. This period is added to the subjective travel time and is weighted with respect to the optimal time E_{opt} as well. Let function $f_{\text{next}}(u,v,t)$ calculate the departure time of the next trip from u to v after time t[1] and $f_{\text{wait}} : T \to T$ a weight function for the waiting period. We get the normalized extended subjective travel time

$$E_{\text{ext}}(u,v,t) := \frac{f_{\text{wait}}(f_{\text{next}}(u,v,t) - t) + E_{\text{subj\_min}}(u,v,f_{\text{next}}(u,v,t))}{E_{\text{opt}}(u,v,t)}$$

We evaluate a route (for the fixed period of time) as the weighted mean of E_{ext} (weighted with the expected number of passengers). So we get the *expected evaluation of a trip* for route u to v, and therefore we can even compare to (single) trips.

$$E_{\text{route}}(u,v) := \frac{\int_{start}^{end} D_{u,v}(t) \cdot E_{\text{ext}}(u,v,t)\mathrm{d}t}{\int_{start}^{end} D_{u,v}(t)\mathrm{d}t}$$

[1]The time calculated may be after the end of the fixed period *start* to *end*.

Choosing a linear approach for function $f_{\text{wait}}(\Delta t) = \gamma_{\text{wait}} \cdot \Delta t$ the parameter γ_{wait} indicates passenger's knowledge of the time table. A low value ($0 \leq \gamma_{\text{wait}} \ll 1$) can be used if many passengers actually know the departure times and will come just in time, a higher value ($0 \ll \gamma_{\text{wait}} \leq 1$) indicates that people are arriving more randomly.[2]

Now we take a closer look to the problem of *which is the next trip*. Paying attention only to pure travel time it's as simple as this: A passenger wouldn't start at time t if he can also leave at time $t' > t$ provided the same arrival time is guaranteed. Hence, "a passenger departs as late as possible provided he will arrive as early as possible".

Pairs of departure and arrival times (of valid trips) form a complete partial order with

$$(t_d, t_a) \leq (t'_d, t'_a) \text{ iff. } t_d \geq t'_d \text{ and } t_a \leq t'_a$$

The minimal elements (with departure times within the period of time *start* to *end*) of this complete partial order correspond (in lexicographical order) to the trips to be chosen in E_{route} (for every non-minimal element there is an element which features both a departure time that is at least as late and an arrival time that is at most as late, hence, the trip corresponding to a non-minimal element wouldn't be chosen; for every trip corresponding to a minimal element holds that there's no other trip that features a later departure time while providing at least the same arrival time because this would result in a smaller element, a contradiction).

Switching to (subjective) travel times (denoted by t_t) we get using

$$(t_d, t_t) \leq (t'_d, t'_t) \text{ iff. } t_d \geq t'_d \text{ and } t_d + t_t \leq t'_d + t'_t$$

a defined (and reasonable) way to choose the next trip.

3.3 Evaluation of a public transport system

We can now evaluate a public transport system (from passenger's point of view). One may want to restrict to a subset $R \subseteq S \times S$ of routes to be optimized, e. g. to concentrate on the main routes or to decrease computer time.

We present three examples for concrete definition of $E_{\text{all}}^{(R)}$:

(1) To optimize the expected contentedness of a passenger choose

$$E_{\text{all}}^{(R)} := \frac{\sum_{(u,v)\in R} \int_{start}^{end} D_{u,v}(t)\mathrm{d}t \cdot E_{\text{route}}(u,v)}{\sum_{(u,v)\in R} \int_{start}^{end} D_{u,v}(t)\mathrm{d}t}$$

(2) To reduce the number of bad routes you may define ($p > 1$)

$$E_{\text{all}}^{(R)} := \frac{\sum_{(u,v)\in R} \int_{start}^{end} D_{u,v}(t)\mathrm{d}t \cdot E_{\text{route}}(u,v)^p}{\sum_{(u,v)\in R} \int_{start}^{end} D_{u,v}(t)\mathrm{d}t \cdot E_{\text{route}}(u,v)^{p-1}}$$

[2] Values > 1 can be of use as well. When optimizing with high values of γ_{wait} we expect (almost) constant intervals between the departure times of trams for this route. The subjective travel times, however, may deteriorate.

This can also be read as weighted sum of E_{route} with weights $\int_{start}^{end} D_{u,v}(t)\mathrm{d}t \cdot E_{\text{route}}(u,v)^{p-1}$.

(3) Computing $\lim_{p\to\infty}$ we get minimization of the worst route

$$E_{\text{all}}^{(R)} := \max_{(u,v)\in R} E_{\text{route}}(u,v)$$

These three examples allow comparison of different public transportation services, comparison to single trips, and between arbitrary subsets R, R' of routes.

3.4 Cost-benefit analysis

The provider has to keep down costs, i. e., he tries to get the passengers being as content as possible while running as few trams, busses, etc. as practicable. [SK94] gives a detailed overview of different costs for providers.

In a cost-benefit analysis we have to consider for example

- additional costs for the provider and additional gains due to more passengers
- and higher volume of traffic resulting from people changing from car to public transport.

Given a time table, it is possible to calculate the minimal number of persons, trams, busses, etc. needed for implementation and to establish a work plan, replacement, periods for maintenance etc. From this you can estimate the operating costs of the transport company and with the assumed volume of traffic the fares (here denoted by f_C) can be fixed. Together with the function $E_{\text{all}}^{(R)}$ we can value a timetable with respect to the actual (or virtual) condition of a transport system.

But in addition, other aspects like competition and subsidies must be kept in mind. Exact computations are no longer possible or will be difficult; this will be done in future research. In a first approach one may fix the provider costs which seems to be a useful assumption.

4 An example with a periodic time table

The provider wants periodic time tables for various reasons (easy to keep in mind for customers, easier train schedule, etc.). In this example we focus on a specific time of day (e. g. rush hour), compute an optimal time table for this period and determine the time table of other times afterwards.[3]

We denote with *line interval* the time between consecutive departure times of a line. Given a periodic time table with (fixed) line intervals $\tau_1, \dots, \tau_{|\mathcal{L}|}$ the time table will be repeated after the least common multiple of $\tau_1, \dots, \tau_{|\mathcal{L}|}$ – this will be used as the time interval for optimization. The expected number of passengers is normally given as

[3]The Stuttgarter Straßenbahnen AG pursues this approach.

an aggregated amount, we assume that people come to the stops uniformly. With fixed line intervals we have just small oscillation of the number of trams/busses needed (the difference is in fact at most one vehicle per line), so we can approximate costs to be constant (alternatively the number of trams/busses needed can be suitably weighted and added to the evaluation).

Let $D_{u,v}$ be the expected number of persons per time unit for route u to v, $E_{\text{opt}}(u,v)$ the optimal travel time for this route and τ_{tt} the time after which the time table is repeated. We get the following evaluations (choosing example (1) from above):

$$\begin{aligned}
E_{\text{all}} := E_{\text{all}}^{S\times S} &:= \frac{\sum_{(u,v)\in S\times S} \tau_{tt} \cdot D_{u,v} \cdot E_{\text{route}}(u,v)}{\sum_{(u,v)\in S\times S} \tau_{tt} \cdot D_{u,v}} \\
E_{\text{route}}(u,v) &:= \frac{\int_{0\le t<\tau_{tt}} D_{u,v} \cdot E_{\text{ext}}(u,v,t)\mathrm{dt}}{\tau_{tt} \cdot D_{u,v}} \\
E_{\text{ext}}(u,v,t) &:= \frac{\gamma_{\text{wait}} \cdot (f_{\text{next}}(u,v,t) - t) + E_{\text{subj_min}}(u,v,f_{\text{next}}(u,v,t))}{E_{\text{opt}}(u,v)}
\end{aligned}$$

Let $t_{uv_1}, \ldots, t_{uv_{n_{uv}}}$ be the departure times of the trips from u to v (let $t_{uv_0} = t_{uv_{n_{uv}}} - \tau_{tt}$), then we get (after some transformation)

$$\begin{aligned}
E_{\text{all}} := &\frac{1}{\sum_{(u,v)\in S\times S} \tau_{tt} \cdot D_{u,v}} \\
&\cdot \left(\sum_{(u,v)\in S\times S} \frac{D_{u,v}}{E_{\text{opt}}(u,v)} \sum_{i=1}^{n_{uv}} \left(\frac{\gamma_{\text{wait}}}{2} \cdot \left(t_{uv_i} - t_{uv_{i-1}}\right)^2 \right.\right. \\
&\qquad \left.\left. + \left(t_{uv_i} - t_{uv_{i-1}}\right) \cdot E_{\text{subj_min}}\left(u,v,t_{uv_i}\right)\right)\right)
\end{aligned}$$

With a given additive weight function for $E_{\text{subj\_min}}$ we can use the Dijkstra algorithm to compute the trips (and departure times). With an adapted version of Dijkstra's algorithm due to [Möh97] we can compute them in $\mathcal{O}(|S|\tau_{tt}\log(|S|\tau_{tt}))$ per source stop (taking into account that the graph built from a consistent time table will obviously be sparse because only edges of neighbouring stops have to be considered there – the time table has to be given accordingly). This results (including costs for sorting the departure times per route and for adding up) in $\mathcal{O}(|S|^2\tau_{tt}\log(|S|\tau_{tt}))$ for computing E_{all}. We can now try to use genetic algorithms or heuristics to get a better time table.

5 Conclusions

We developed comprehensible objective functions for time table optimization. Evaluations are meaningful and allow the comparison between different systems like busses versus trams or transport system versus single trips.

The proposed evaluations can be extended for intermodal transport, i.e. integration of public and private traffic. The models should be calibrated in a way that allows the comparison between different means of transport.

Further research is needed for including evaluations in cost-benefit analysis. Moreover it's desirable to have an integrated approach to line optimization and time table optimization, so we get line plans that allow "good" time tables.

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